

(1) 20%

Let $\beta(s)$ be the unit-speed curve, $\beta(s) = (\frac{4}{5} \cos s, 1 - \sin s, -\frac{3}{5} \cos s)$.

(a) Compute the Frenet frame T, N, B and the curvature and torsion functions of $\beta(s)$.

(b) Show that curve $\beta(s)$ is a circle; and find its center and radius.

(2) 20%

Find two regular surfaces with Gauss curvature $\equiv 0$ and different second fundamental forms.

(3) 20%

(a) Give the suitable definitions of Gauss map and principal curve.

(b) Prove or disprove the following:

Let surface S with Gauss curvature $K = 1$, then $S = S^2$? Where S^2 is a unit sphere.

(4) 20%

Let S be a regular closed surface of diameter 1 and $k_1(x), k_2(x)$ be the principal curvatures at $x \in S$ with $k_1(x) \geq k_2(x)$. Show that

$$\max_{x \in S} \{k_1(x)\} \geq 1.$$

(5) 20%

Let $y = f(x), -\infty < x < \infty$ be a smooth curve. Let S be a regular minimal surface rotating the curve $f(x)$ around x -axis. Determine $f(x)$.