Each question has 10 points. Please answer all questions.

1. Let $X = [0,1]$ be the closed unit interval (under the usual topology). Show that if $f : X \longrightarrow X$ is a continuous function, then there is an $x \in X$ such that $f(x) = x$.

2. Give 4 different topologies on the set $A = \{a, b, c, d, e, f\}$ so that no 2 of them are homeomorphic.

3. Let $f : X \longrightarrow Y$ be a continuous function from a compact space X to a metric space Y. Show that $f(X)$ is bounded in Y.

4. Let E be a connected subset of a Hausdorff space X. Suppose E contains more than one element. Show that E is infinite.

5. Let E be a non-empty open connected subset of the plane R^2 (under the usual topology). Show that E is path-connected.

6. A space is totally discrete if its only connected subsets are one-points. Is it true that if X has the discrete topology, then X is totally discrete? Does its converse hold?

7. Use the fact that the real line R is uncountable to show: if $A \subset R^2$ is a countable set, then $R^2 \setminus A$ is connected.

8. Give examples of an open map f, and a closed map g from the real line R (under the usual topology) to itself so that both f, g are not homeomorphism from R to R.

9. Let $R^n, n \geq 1$ be the Euclidean space which has the usual metric. Show that

$$S^n = \{(x_1, x_2, \cdots, x_n) \in R^n \mid x_1^2 + x_2^2 + \cdots + x_n^2 = 1\}$$

is a closed set in R^n.

10. Show that under the usual topology, R^n and R are not homeomorphic if $n > 1$.