Part I. Prove or disprove the following statements. (60%)

1. Let $A$ be any set in $\mathbb{R}^2$, and $B$ be the set of all cluster points of $A$. Then $B$ is a closed set.

2. Let $z_n$ be a sequence of complex numbers with $\lim z_n = A$, then
   \[ \lim \frac{z_1 + z_2 + \ldots + z_n}{n} = A \]

3. Let $r = \frac{1}{1!} + \frac{1}{10!} + \frac{1}{(10^2)!} + \frac{1}{(10^3)!} + \ldots + \frac{1}{(10^n)!} + \ldots$, then $r$ is an irrational number.

4. Let $\{f_n(x)\}$ be a sequence of differentiable functions defined in $(0, 1)$ with the property that $f_n(x) \to f(x)$ uniformly on $(0, 1)$. Then $\frac{d}{dx} f_n(x) \to \frac{d}{dx} f(x)$.

5. Let $\{f_n\}$ be a sequence of Riemann integrable functions defined on $(0, 1)$, with $f_n \to f$ uniformly on $(0, 1)$. Then $f(x)$ is Riemann integrable on $(0, 1)$.

6. Let $a_n = \frac{1}{3 - a_n}$, $n = 1, 2, 3, \ldots$. If $0 < a_1 \leq 2$, then $\{a_n\}$ is a convergent sequence.

Part II (You need to give a detail proof) (40%)

1. (a) Show that a continuous function is Riemann integrable in any closed intervals.
   
   (b) Define the function $f : [0, 1] \to \mathbb{R}$ by $f(x) = \frac{\sin x}{x^n}$, if $x \neq 0$, and $f(0) = 0$.

   For what value of integers $n$ is $f(x)$ Riemann integrable? Justify your answer.

2. Let $f(x)$ be a continuous function defined on $(0, 1)$. Show that $\lim_{x \to 0^+} f(x)$ and $\lim_{x \to 1^-} f(x)$ both exist if and only if $f(x)$ is a uniformly continuous function.