

共五題，總分100分

ADVANCED CALCULUS

This exam contains 5 problems with a total of 100 points. Each problem costs 20 points. Do all problems and show all your work for partial credits.

1. Compute the improper integral:

$$\int_0^{\infty} e^{-9x^2+1} dx.$$

2. Suppose that  $\sum_{n=1}^{\infty} a_n(x-a)^n$  converges uniformly for  $|x-a| < h$ . Is it true that it converges uniformly for  $|x-a| \leq h$ ? Give your reasons.

3. Find the Taylor series of the function  $f(x) = \sin^2 x$  at  $x = 0$  and find the maximal connected interval in  $\mathbb{R}$  where it converges uniformly.

4. Find the maximal value of the function  $f(x, y) = xy + 2x$  where  $(x, y)$  lies in the ellipse  $x^2 + 4y^2 = 4$ .

5. Suppose that all first partial derivatives of a function  $f: \mathbb{R}^n \rightarrow \mathbb{R}$  are continuous at a point  $a$ . Show that  $f$  is differentiable at  $a$ .