

Advanced Calculus

This exam contains 5 problems with a total of 100 points. Each problem costs 20 points. Do all problems and show all your work for partial credits.

1. Compute the improper integrals :

$$(1) \int_0^1 \ln x \, dx \qquad (2) \int_0^2 \frac{1}{\sqrt{2x-x^2}} \, dx.$$

2. Suppose that the series $f(x) = \sum_{n=0}^{\infty} a_n(x-a)^n$ converges for $|x-a| < h$. Is it true that $f(x)$ must be continuous? Give your reasons.

3. Find the Taylor polynomial of degree 3 in powers of x and y for e^{3x-xy} .

4. Show that the equation $\sin(x+2y) = 2x(y+1)$ has a solution of the form $y = f(x)$ near $x = 0$ with $f(0) = 0$ and find $f'(0)$.

5. Find the shortest distance from the origin to the curve $x^2y = 128$.