

Answer all questions.

1. (15 pts.) Let $f(x) = x^2$ when x is irrational and $f(x) = 0$ when x is rational.
a) Show that f is continuous at $x = 0$.
b) Show that f is differentiable at $x = 0$. What is $f'(0)$?

2. (10 pts.) Show that the function

$$\sum_{n=0}^{\infty} n \frac{n^2 + 1}{(n + 10)^2} x^n$$

is well defined and continuous for $x \in (-1, 1)$.

3. (10 pts.) Let $g(x, y) = 2xy - 3x^2$, $h(x, y) = x^2 + y$. Is there a function $F(x, y)$ so that

$$\left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y} \right) = (g, h)$$

Compute $F(x, y)$ if you say yes.

4. (10 pts.) Find the area of the sphere $x^2 + y^2 + z^2 = 16$ inside the paraboloid $2z = x^2 + y^2$.

5. (15 pts.) Transform and simplify

$$\iint_D f(xy) \, dx \, dy$$

where D is the region in the first quadrant bounded by $xy = 1$, $xy = 2$, $x = y$, $y = 4x$, by the transformation $x = u/v$, $y = v$. Compute the integral with $f(xy) = (xy)^{10}$.

6. (15 pts.) Consider the pair of equations

$$u^2 + 2xu - v^2 - 7y = 0$$

$$u^3 + 3 \sin u - 15e^{x-v} + \cos v + 14 = 0$$

Show that x and y can be solved in terms of u and v in a neighborhood of $x = y = u = v = 0$. Compute

$$\frac{\partial x}{\partial u}(0, 0) \quad \text{and} \quad \frac{\partial y}{\partial v}(0, 0)$$

7. (10 pts.) Show that $x^3 + x^2 + 6x + 1 = 0$ has one and only one real root.

8. (15 pts.) Find a power series solution to $x'(t) = 1 + tx(t)$, $x(0) = 0$. What is the radius of convergence of your solution $x(t)$.