Answer all questions. There are 10 problems.

1. (10 pts) (a) Show that the equation \( x^n + ax + b = 0 \), \( n \) an even positive integer, has at most 2 distinct real roots.
(b) Show that the equation \( x^{10} - 10x + 1 = 0 \) has exactly 2 distinct real roots.

2. (10 pts) Suppose \( f \) is continuous and increasing on \([0,1]\). Prove that
\[
\sup f(E) = f(\sup E)
\]
for every non-empty set \( E \subset [0,1] \).

3. (10 pts) Suppose \( E \subset \mathbb{R} \) is compact and non-empty. Prove that \( \sup E, \inf E \in E \).

4. (10 pts) (a) Prove that the function \( f(x) = x^2 \ln x \) is uniformly continuous on \((0,1)\).
(b) Prove that the function \( g(x) = x^2 \) is not uniformly continuous on \((0,\infty)\).

5. (10 pts) Suppose \( f \) is continuously differentiable and 1-1 on \([a,b]\). Prove that
\[
\int_a^b f(x) \, dx + \int_{f(a)}^{f(b)} f^{-1}(x) \, dx = b(f(b)) - a(f(a)).
\]

6. (10 pts) Prove that \( 1/(xn) \to 0 \) pointwise, but not uniformly on \((0,1)\) as \( n \to \infty \).

7. (10 pts) (a) Show that \( \sin t \leq t \) for all \( t > 0 \).
(b) Show that \( |\sin t| \leq |t| \) for all \( t \).
(c) Show that
\[
f(x) = \sum_{k=1}^{\infty} \frac{1}{k} \sin \left( \frac{x}{k+1} \right)
\]
converges uniformly on \([-1,1]\).

8. (10 pts) (a) Let \( f(0,0) = 0 \) and
\[
f(x, y) = \frac{x^2 y}{x^4 + y^2} \quad \text{if} \quad (x, y) \neq (0,0).
\]
Prove that \( f(x, y) \) is not continuous at \((0,0)\).
(b) Let \( g(0,0) = 0 \) and
\[
g(x, y) = \frac{x^2 y}{x^2 + y^2} \quad \text{if} \quad (x, y) \neq (0,0).
\]
Prove that \( g(x, y) \) is continuous at \((0,0)\).

9. (10 pts) Compute
\[
\int \int_E \sqrt{x-y} \sqrt{x+2y} \, dA,
\]
where \( E \) is the parallelogram with vertices \((0,0), (2/3,-1/3), (1,0), (1/3,1/3)\).

10. (10 pts) (a) Prove that there is a function \( g(s,t) \), continuously differentiable on some open set \( V \) containing the point \((1,0)\), such that \( g(1,0) = 1 \) and
\[
sx^2 + tx^3 + 2\sqrt{s} + s + t^2 x^4 - x^5 \cos t - x^6 = 1
\]
for \( x = g(s,t) \) and \((s,t) \in V\).
(b) Compute
\[
\frac{\partial g}{\partial s}(1,0) \quad \text{and} \quad \frac{\partial g}{\partial t}(1,0).
\]