This exam has 5 questions, for a total of 100 points.

1. (20 points) Let $f : R^2 \to R$ be defined by

$$f(x,y) = x^3 + xy + y^3 + 1.$$

For which points $p = (0,0), p = (\frac{2}{3}, \frac{2}{3})$ is $f^{-1}(f(p))$ an imbedded submanifold in R^2?

2. Prove or disprove the following.
   (a) (10 points) Let M be an n-dimensional compact manifold, and let $f : M \to R^2$ be smooth, then f can not everywhere be nonsingular.?
   (b) (10 points) Let G be a Lie group with Lie algebra $\wp$. If $X, Y \in \wp$, then for t sufficiently small, then $\exp tX \exp tY = \exp\,(t(X+Y))$?

3. Let M be a compact Riemannian 2 manifold, and K be its Gaussian curvature.
   (a) (10 points) If $K > 0$, then the first fundamental group, $\pi_1(M)$ is finite.
   (b) (10 points) If $K \leq 0$, then M has genus at least one.

4. (20 points) Suppose M_1 and M_2 are two Riemannian manifolds and $f_i : M_1 \to M_2$ are Riemannian isometries that converge uniformly to a map $f : M_1 \to M_2$(it means, for any $\epsilon > 0$, there exists N such that $d(f_i(p), f(p)) < \epsilon$ for all $p \in M$ and $i \geq N$), then show that f is a Riemannian isometry.

5. (20 points) Let M_1, M_2 be two Riemannian manifolds. Suppose that $f : M_1 \to M_2$ is a smooth covering map, that is also local isometry. If M_1 is complete, then M_2 is also complete. (or M_1 is complete, then M_1 is also complete)