

1. (10 points) Let

$$f(x) = \frac{1}{\sin x} - \frac{1}{x}.$$

Find  $\lim_{x \rightarrow 0} f(x)$ .

2. (10 points) Compute the area of the infinite region between the curve  $\frac{1}{1+x^2}$  and the  $x$ -axis.

3. (15 points) Find the absolute maximum and minimum values of  $f(x) = x^{2/3}$  on the interval  $-8 \leq x \leq 27$ .

4. Determine whether the following series converges or diverges.

a. (10 points)

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

b. (15 points)

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2}$$

5. (10 points) Prove or disprove the following statement. If  $A$  and  $B$  are two  $n \times n$  matrices with  $n > 1$ , then  $\det(A + B) = \det(A) + \det(B)$ .

6. (10 points) Let  $u$  be an  $n \times 1$  nonzero vector, where  $n > 1$  and  $u^T$  be the transpose of  $u$ , find the eigenvalues of the matrix  $uu^T$  and its determinant.

7. (20 points) Find two variables  $u$  and  $v$  and two values  $\lambda_1$  and  $\lambda_2$  such that

$$5x^2 - 2xy + 5y^2 = 4$$

can be rewritten as

$$\lambda_1 u^2 + \lambda_2 v^2 = 4,$$

where  $u = ax + by$  and  $v = cx + dy$  with  $a, b, c, d \in \mathbb{R}$ .