1. (10 points) Let

\[ f(x) = \frac{1}{\sin x} - \frac{1}{x}. \]

Find \( \lim_{x \to 0} f(x) \).

2. (10 points) Compute the area of the infinite region between the curve \( \frac{1}{1+x^2} \) and the \( x \)-axis.

3. (15 points) Find the absolute maximum and minimum values of \( f(x) = x^{2/3} \) on the interval \(-8 \leq x \leq 27\).

4. Determine whether the following series converges or diverges.
   a. (10 points)

   \[ \sum_{n=1}^{\infty} \frac{1}{n} \]

   b. (15 points)

   \[ \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2} \]

5. (10 points) Prove or disprove the following statement. If \( A \) and \( B \) are two \( n \times n \) matrices with \( n > 1 \), then \( \det(A + B) = \det(A) + \det(B) \).

6. (10 points) Let \( u \) be an \( n \times 1 \) nonzero vector, where \( n > 1 \) and \( u^T \) be the transpose of \( u \), find the eigenvalues of the matrix \( uu^T \) and its determinant.

7. (20 points) Find two variables \( u \) and \( v \) and two values \( \lambda_1 \) and \( \lambda_2 \) such that

\[ 5x^2 - 2xy + 5y^2 = 4 \]

can be rewritten as

\[ \lambda_1 u^2 + \lambda_2 v^2 = 4, \]

where \( u = ax + by \) and \( v = cx + dy \) with \( a, b, c, d \in \mathbb{R} \).