1. (10%) Suppose that, on average, a post office handles 10,000 letters a day with a variance of 2000. What can be said about the probability that this post office will handle between 8000 and 12,000 tomorrow? [Hint: Use Chebyshev's inequality.]

2. (10%) The coefficient of the quadratic equation \( ax^2 + bx + c = 0 \) are determined by tossing a fair die three times (the first outcome is \( a \), the second one \( b \), and the third one \( c \)). Find the probability that the equation has no real roots.

3. (12%) Suppose that \( \{ E_n, \ n \geq 1 \} \) is either an increasing or a decreasing sequence of events. Show that

\[
\lim_{n \to \infty} p(E_n) = p(\lim_{n \to \infty} E_n)
\]

4. (12%) Show that if all three of \( n, N, \) and \( D \to \infty \), so that \( n/N \to 0 \), \( D/N \) converges to a small number, and \( nD/N \to \lambda \), then for all \( x \),

\[
\binom{D}{x} \binom{N-D}{n-x} \frac{e^{-\lambda} \lambda^x}{x!} \to \frac{e^{-\lambda} \lambda^x}{x!}
\]

5. (16%) Imagine a population of \( N + 1 \) urns. Urn number \( k \) contains \( k \) red, \( N - k \) white \( \{ 0, 1, \ldots, N \} \) balls. An urn is chosen at random and \( n \) random drawings are made from it, the ball drawn being replaced each time. Define

- Event A: All \( n \) balls turn out to be red,
- Event B: The \( (n+1) \)st draw yields a red ball.

(a) Find \( P(A| \text{Urn } k \text{ is chosen}) \) \( (k = 0, 1, \ldots, N) \).
(b) Find \( P(A) \).
(c) Find \( P(AB) \).
(d) Find \( P(B|A) \).
6. (15%) Let $X_1$ and $X_2$ be independent r.v.'s, each $N(\mu, \sigma^2)$. Let $Y = X_1 + X_2$, $Z = X_1 - X_2$. Show that $Y$ and $Z$ are independent r.v.'s. [Hint: Use transformations to find the joint p.d.f. of $(Y, Z)$, and then the independence.]

7. (10%) Prove that (for any fixed $\lambda > 0$)

$$p_x (x) = \begin{cases} e^{-\lambda} \frac{\lambda^x}{x!}, & x = 0, 1, 2, \ldots \\ 0, & \text{otherwise} \end{cases}$$

is a probability function.

8. (15%) The joint probability mass function of $X$ and $Y$ is given by

$$P (1, 0) = \frac{1}{6} \quad P (1, 1) = \frac{1}{6}$$

$$P (2, 0) = \frac{1}{6} \quad P (2, 1) = \frac{1}{3} \quad P (2, 2) = \frac{1}{6}$$

(a) Compute the conditional mass function of $Y$ given $X=i$, $i=1,2$.

(b) Find $E(Y|X)$, $\text{VAR}(E(Y|X))$ and $E(\text{VAR}(Y|X))$. 