國立中正大學九十二學年度碩士班招生考試試題

系所別：統計科學研究所　　　　科　目：基礎數學

Total 7 problems

(15%) 1. Let $T(x, y, z) = 20 + 2x + 2y + z^2$ represent the temperature at each point on the sphere $x^2 + y^2 + z^2 = 11$. Use Lagrange multipliers to find the extreme temperatures on the curve formed by the intersection of the plane $x + y + z = 3$ and the sphere.

(15%) 2. Let $R = \left\{(x, y, z) \in R^3 \mid \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1\right\}$. Evaluate the integral

$$\iiint_R xyz\,dx\,dx\,dz.$$

3.

(10%)(a) Find the series representation of e^{-x^2} and determine its interval of convergence.

(5%)(b) Use a power series to approximate $\int_0^1 e^{-x^2}dx$ with an error of less than 0.01.

(10%) 4. Find the derivative of the function

$$F(x) = \frac{[x^2]}{1 + x^2}$$

whenever it exists. ($[x]$ is the greatest integer function)

(15%) 5. Find an orthogonal matrix P and a diagonal matrix D such that $P^{-1}AP = D$ where $A = \begin{bmatrix} 8 & -2 & 2 \\ -2 & 5 & 4 \\ 2 & 4 & 5 \end{bmatrix}$.

(10%) 6. A matrix P is idempotent if $P^2 = P$. If P is symmetric, then P is idempotent and of rank r if and only if it has r eigenvalues equal to unity and $n - r$ eigenvalues equal to zero.

(20%) 7. Let T be the linear operator on R^2 define by

$$T(x_1, x_2) = (-x_2, x_1).$$

(a) What is the matrix of T in the standard ordered basis for R^2?

(b) What is the matrix of T in the ordered basis $B = \{(1, 2), (1, -1)\}$?

(c) Prove that for every real number c the operator $(T - cI)$ is invertible.

(d) Prove that if B is any ordered basis for R^2 and $[T]_B = A$, then $a_{12}a_{21} \neq 0$.