

國立中正大學九十一學年度學士班二年級轉學生招生考試試題

學系別：數學、物理、化學、資訊工程、
機械工程、經濟、資訊管理學系

科目：微積分

第 1 頁，共 2 頁

★ There are no partial credits for Problems 1-6. Do not show your work!

1. (8 points) Evaluate the following limits.

(a) $\lim_{x \rightarrow \pi} \frac{\sin x}{x^2 - \pi^2}$

(b) $\lim_{x \rightarrow 1} x^{1/1-x}$

2. (4 points) Evaluate $\frac{d}{dx} \int_{x^2}^{x^3} \frac{dt}{\sqrt{t^3 + 1}}$.

3. (12 points) Evaluate the following integrals.

(a) $\int_0^{\frac{\pi}{2}} \frac{d}{dx} \left(\sin \frac{x}{2} \cos \frac{x}{3} \right) dx$

(b) $\int \frac{dt}{e^t + e^{-t}}$

(c) $\int x^2 \cos 3x dx$

4. (12 points) Determine whether the following series are divergent, absolutely convergent or conditionally convergent.

(a) $\sum_{n=1}^{\infty} \ln\left(\frac{2n}{3n+2}\right)$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

(c) $\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^2}$

(d) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n-1}}{n}$

5. (8 points) Set up the integral for the volume of the solid obtained by rotating about the y -axis the region between $y = x$ and $y = x^2$, by using (a) the slicing method and (b) cylindrical shells.

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第2頁，共2頁

6. (8 points) Sketch the graph of $f(x)$ satisfying the following conditions: $f(0) = 0$, $f'(-2) = f'(1) = f'(3) = 0$, $\lim_{x \rightarrow \infty} f(x) = 0$, $\lim_{x \rightarrow 2} f(x) = -\infty$, $\lim_{x \rightarrow -\infty} f(x) = \infty$, $f'(x) < 0$ on $(-\infty, -2)$, $(1, 2)$ and $(3, \infty)$, $f'(x) > 0$ on $(-2, 1)$ and $(2, 3)$, $f''(x) > 0$ on $(-\infty, 0)$ and $(5, \infty)$ and $f''(x) < 0$ on $(0, 2)$ and $(2, 5)$. Show clearly the convexity and the asymptotes of your graph.

★ For the next problems you need to show all your work.

7. (6 points) Find the absolute maximal and minimal values, if any, of the function

$$f(x) = x^3 - 3x^2 - 1, \quad -\frac{1}{2} \leq x \leq 4.$$

8. (8 points) The half-life of radium-226 is 1590 years. A sample of radium-226 has a mass of 250 mg. Give the differential equation that describes the exponential decay of the mass of radium-226 and use it to find a formula for the mass of radium-226 that remains after t years.

9. (8 points) Evaluate $\int e^{-x^2} dx$ as an infinite series and determine its radius of convergence.

10. (6 points) If $x + y^2 + z^3 + 2xyz = 1$, find $\left. \frac{\partial^2 z}{\partial x^2} \right|_{(0,0,1)}$.

11. (8 points) Approximate $f(1.97, 1.06)$ using linear approximation where $f(x) = \sqrt{20 - x^2 - 7y^2}$.

12. (12 points) Let $\mathbf{F}(x, y) = (-yi + xj)/(x^2 + y^2)$. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is an arbitrary simple closed path which encloses the origin clockwise.