There are no partial credits for Problems 1–6. Do not show your work!

1. (8 points) Evaluate the following limits.

(a) \[ \lim_{x \to x} \frac{\sin x}{x^2 - \pi^2} \]

(b) \[ \lim_{x \to 1} x^{1/1-x} \]

2. (4 points) Evaluate \[ \frac{d}{dx} \int_{x^2}^{3} \frac{dt}{\sqrt{t^3 + 1}}. \]

3. (12 points) Evaluate the following integrals.

(a) \[ \int_{0}^{\frac{\pi}{2}} \frac{d}{dx} \left( \frac{\sin x}{2} \cdot \frac{x}{3} \right) \, dx \]

(b) \[ \int \frac{dt}{e^t + e^{-t}} \]

(c) \[ \int x^2 \cos 3x \, dx \]

4. (12 points) Determine whether the following series are divergent, absolutely convergent or conditionally convergent.

(a) \[ \sum_{n=1}^{\infty} \ln \left( \frac{2n}{3n + 2} \right) \]

(b) \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n} + 1} \]

(c) \[ \sum_{n=1}^{\infty} \frac{1}{n(\ln n)^2} \]

(d) \[ \sum_{n=1}^{\infty} \frac{\sqrt{n + 1} - \sqrt{n - 1}}{n} \]

5. (8 points) Set up the integral for the volume of the solid obtained by rotating about the \( y \)-axis the region between \( y = x \) and \( y = x^2 \), by using (a) the slicing method and (b) cylindrical shells.
6. (8 points) Sketch the graph of \( f(x) \) satisfying the following conditions: \( f(0) = 0 \), \( f'(-2) = f'(1) = f'(3) = 0 \), \( \lim_{x \to \infty} f(x) = 0 \), \( \lim_{x \to -2} f(x) = -\infty \), \( \lim_{x \to -\infty} f(x) = \infty \), \( f'(x) < 0 \) on \((-\infty, -2)\), \((1, 2)\) and \((3, \infty)\), \( f'(x) > 0 \) on \((-2, 1)\) and \((2, 3)\), \( f''(x) > 0 \) on \((-\infty, 0)\) and \((5, \infty)\) and \( f''(x) < 0 \) on \((0, 2)\) and \((2, 5)\). Show clearly the convexity and the asymptotes of your graph.

★ For the next problems you need to show all your work.

7. (6 points) Find the absolute maximal and minimal values, if any, of the function

\[
f(x) = x^3 - 3x^2 - 1, \quad -\frac{1}{2} \leq x \leq 4.
\]

8. (8 points) The half-life of radium-226 is 1590 years. A sample of radium-226 has a mass of 250 mg. Give the differential equation that describes the exponential decay of the mass of radium-226 and use it to find a formula for the mass of radium-226 that remains after \( t \) years.

9. (8 points) Evaluate \( \int e^{-x^2} \, dx \) as an infinite series and determine its radius of convergence.

10. (6 points) If \( x + y^2 + z^3 + 2xyz = 1 \), find \( \frac{\partial^2 z}{\partial x^2} \bigg|_{(0,0,1)} \).

11. (8 points) Approximate \( f(1.97, 1.06) \) using linear approximation where \( f(x) = \sqrt{20 - x^2 - 7y^2} \).

12. (12 points) Let \( F(x, y) = (-y1 + xj)/(x^2 + y^2) \). Find \( \int_C F \cdot dr \) where \( C \) is an arbitrary simple closed path which encloses the origin clockwise.