Calculus

Part I. No partial credit will be given in this part. Do not need to show all your work. Only the **final result** will be needed.

(1) (6 points) Find all points on the graph of the function $f(x) = 2\sin x + \sin^2 x$, $0 \leq x < 2\pi$, at which the tangent line is horizontal.

(2) (6 points) A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a constant speed of 15 ft/s. When he passes under the balloon it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 seconds later?

(3) (6 points) Find the local maximum and minimum values of the function $f(x) = x^{1/3}(x+3)^{2/3}$.

(4) (6 points) Evaluate $\lim_{n\to\infty}\left(\sum_{i=1}^{n}\frac{1}{n+(i^2/n)}\right)$.

(5) (6 points) The marginal cost of manufacturing x yards of a certain fabric is $C'(x) = 3 - 0.01x + 0.000006x^2$ (in dollars per yard). Find the increase in cost if the production level is raised from 2000 yards to 4000 yards.

(6) (6 points) If $x\sin \pi x = \int_0^{x^2} f(t)\,dt$, where f is a continuous function, find $f(4)$.

(7) (6 points) Suppose f is an increasing function, $f'(x) > 0$, $f(0) = 0$, $f(1) = 1$, and $\int_0^1 f(x)\,dx = \frac{1}{3}$. Find the value of the integral $\int_0^1 f^{-1}(y)\,dy$, where f^{-1} is the inverse function of f.

國立中正大學九十二學年度學士班二年級轉學生招生考試試題

學系別：數學、物理、化學、資訊工程、機械工程　　科目：微積分

(8) (6 points) **Set up, but do not evaluate**, an integral for the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$.

(9) (18 points) Evaluate the integral.

(a) $\displaystyle\int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{16 - x^2}}\, dx$　(b) $\displaystyle\int_0^3 \frac{1}{x^2 - x - 2}\, dx$　(c) $\displaystyle\int x^5 e^{-x^3}\, dx$

(10) (6 points) If $f(x, y) = xe^y$, find the rate of change of f at the point $P(2, 0)$ in the direction toward the point $Q(\frac{1}{2}, 2)$. In which direction does f have the maximum rate of change? What is the maximum rate of change?

(11) (6 points) Evaluate the double integral $\displaystyle\iint_D y^3\, dA$, where D is the triangular region with vertices $(0, 2)$, $(1, 1)$, and $(3, 2)$.

Part II. Partial credits will be given in this part. Show **all your work** to get credits.

(12) (8 points) Find the interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty} n^3 x^n$ and find its sum.

(13) (7 points) Show that when Laplace's equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = 0$ is written in cylindrical coordinates (r, θ, z), it becomes $\dfrac{\partial^2 u}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial u}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2 u}{\partial \theta^2} + \dfrac{\partial^2 u}{\partial z^2} = 0$.

(14) (7 points) Evaluate the line integral $\displaystyle\int_C yz\, dx + xz\, dy + xy\, dz$, where C consists of line segments from $(0, 0, 0)$ to $(2, 0, 0)$, from $(2, 0, 0)$ to $(1, 3, -1)$, and from $(1, 3, -1)$ to $(1, 3, 0)$.