

國立中正大學八十六學年度轉學生考試試題

所別：數學系

科目：微積分

國立中正大學圖書館
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Part I. In this part you need not to show your work. There will be no partial credits, so accuracy is very important. Each problem worths 6 points.

1. The equation $x + 3xy - e^y + 1 = 0$ describes a curve passing through the origin. The tangent line of this curve through $(0,0)$ has slope _____.

2. Let $f(x) = \frac{\sqrt{x^4 + 5x + 1}}{(1+x)^2(3+x)^3} e^x$. Then $f'(0) =$ _____.

3. Let $f(x) = \frac{1 - \cos x}{x}$, and $g(x) = f(f(f(x)))$. The $\lim_{x \rightarrow 0} \frac{g(x)}{x} =$ _____.

4. If $w = 3x^2y + 2xyz + z^3y$ with $x = \cos t, y = \sin t, z = e^t$, then $\frac{dw}{dt} \Big|_{t=0} =$ _____.

5. Evaluate the definite integral $\int_0^{\frac{\pi}{2}} (\sin x)^{100} dx =$ _____.

6. Use calculus to estimate the area of the shaded region (as in Fig. 1. page2) The rectangle has length 12 and width 6, and the path has width 0.4 _____.

7. Evaluate the line integral $\int_C (1 + 2xy) dx + x^2 dy =$ _____ . Where C is the circular arc joining $(1,0)$ and $(0,1)$ (as in Fig. 2. page2)

8. The total area of the region in the plane enclosed by the curve $(x^2 + y^2)^2 = 2a^2(x^2 - y^2)$ is _____.

9. The equation of the tangent plane of the sphere $x^2 + y^2 + z^2 = 6$ at the point $(1,1,2)$ is _____.

10. If $f(x) = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \dots + \frac{x^n}{(n+1)!} + \dots$, and $\frac{f(x)}{1-x} = \sum_{n=0}^{\infty} a_n x^n$. Then

$a_n =$ _____.

Part II. In this part you need to show your work to get any credits.

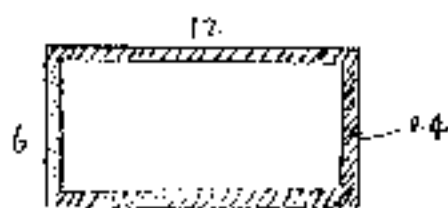
1. (a) A manufacture has been allotted 30,000 dollars to spend on the development and promotion of a new product. It is estimated that if x thousand dollars is spent on development and y thousand dollars is spent on promotion, approximately $f(x, y) = 20x^{\frac{3}{2}}y$ units of product will be sold. How much money should the manufacture allocate to development and how much to promotion in order to maximize sales? What is this maximum?

(b) Suppose the manufacture is allotted 30,200 dollars instead of to spend on the development and promotion of the new product. Estimate how the additional 200 dollars will affect the maximum sales. (14 points)

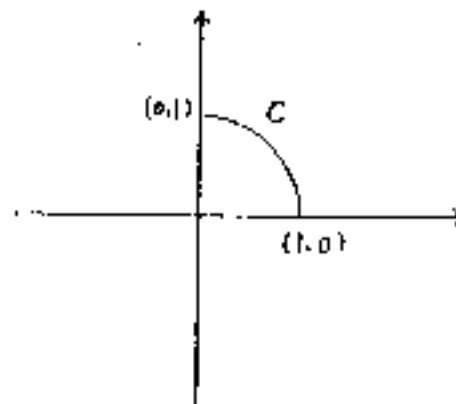
2. Find the area of the part of the paraboloid $z = x^2 + y^2$ inside the sphere $x^2 + y^2 + z^2 = 6$. (12 points)

3. (a) Prove that for all $x \in \mathbb{R}$, we have $|\sin x| \leq |x|$.

(b) Let x_n be any real number, we define a sequence $\{x_n\}$ by $x_n = \sin x_{n-1}$, $n = 1, 2, 3, \dots$. Show that $\lim_{n \rightarrow \infty} x_n = 0$. (14 points)



(Fig 1)



(Fig 2)