Show all your work.

1. (10 pts.) A light is 4 miles from a straight shoreline. The light revolve at the rate of 2 rev/min. Find the speed of the spot of light along the shore when the light spot is 2 miles past the point on the shore closest to the source of light.

2. (10 pts.) Sketch the graph of $f(x) = e^{-x}\sin x$ for $x \geq 0$, and determine as many as possible of the key features such as range, intercepts, relative extrema, inflection points, asymptotes, and concavity.

3. (10 pts.) Minimize $f(x,y,z) = 2x^2 + 3y^2 + 4z^2$ subject to $x + y = 4$ and $x - 2y + 5z = 3$.

4. (10 pts.) Find the equation for the tangent line to the curve

$$y = F(x) = \int_1^{\sqrt{x}} \frac{t^2 + t + 1}{\sqrt{3t^2 + 1}}\,dt \quad \text{at } x = 1.$$

5. (10 pts.) Find the volume of the solid D bounded below by the paraboloid $z = x^2 + y^2$ and above by the plane $2x + z = 3$.

6. (10 pts.) Let $f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$.

   (1) Use the definition of the derivative to prove that f is differentiable at $x = 0$.

   (2) Prove or disprove that $f'(x)$ is continuous at $x = 0$.

7. (10 pts.) Let $a_n = \left(1 + \frac{1}{\text{n}}\right)^n$, $n = 1, 2, 3, \ldots$.

   (1) Show that the sequence $\{a_n\}_{n=1}^{\infty}$ converges.

   (2) Approximate $\lim_{n\to\infty} \left(1 + \frac{1}{\text{n}}\right)^n$ correct to four decimal places.

8. (10 pts.) Let $f(x,y,z) = z(x-y)^5 + xy^2z^3$.

   (1) Find the directional derivative of f at (2, 1, -1) in the direction of the outward normal to the sphere $x^2 + y^2 + z^2 = 6$.

   (2) In what direction is the directional derivative at (2, 1, -1) largest?

9. (20 pts.) Define $F(s) = L\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\,dt$. And, the inverse $L^{-1}\{F(s)\}$ is the function $f(t)$ such that $L\{f(t)\} = F(s)$.

   (1) Show that $L\{e^{at} f(t)\} = F(s-a)$ and $L\{tf(t)\} = -F'(s)$.

   (2) Find $L\{t^n\}$, $L\{\cos at\}$ (with $s - a > 0$), $L\{t\cos 2t\}$, and $L^{-1}\left\{\frac{5}{s}\right\}$.