1. (10pts) Let f be a function defined on the interval $[0,1]$ given by $f(x)=1$ if $x\in\mathbb{Q}$ and $f(x)=-1$ if $x\notin\mathbb{Q}$. Show that f is not Riemann integrable.

2. (15pts) Let $A=\{\frac{1}{n}\mid n\in\mathbb{N}\}$ and $B=\{\frac{1}{n}\mid n\in\mathbb{N}\}\cup\{0\}$. Answer the following questions, and *justify* your answer.

   (a) Is A closed?

   (b) Is B compact?

3. (15pts) Let $f(x)=\sqrt{x}:[0,\infty)\to\mathbb{R}$. Answer the following questions, and *justify* your answer.

   (a) Is f uniformly continuous on $[0,2]$?

   (b) Is f uniformly continuous on $[1,\infty)$?

   (c) Is f uniformly continuous on $[0,\infty)$?

4. (15pts) Let $f_n:[0,1]\to[0,\infty)$ be a sequence of continuous functions pointwise convergent to zero, i.e., $f_n\to 0$ pointwise. Prove or disprove that

$$\lim_{n\to\infty}\int_0^1 f_n(x)dx=\int_0^1\lim_{n\to\infty}f_n(x)dx.$$

5. (15pts) A real-valued function defined on (a,b) is called *convex* when the following inequality holds for $x,y\in(a,b)$ and $t\in[0,1]$:

$$f(tx+(1-t)y)\le tf(x)+(1-t)f(y).$$

   If f has a continuous second derivative and $f''>0$, show that f is *convex*.

6. (15pts) Show that

$$\int_0^\infty\frac{\sin x}{x}dx$$

   is *conditionally but not absolutely* convergent.

7. (15pts) Does the map

$$(x,y)\mapsto\left(\frac{xy}{x^2+y^2},\frac{x^2-y^2}{x^2+y^2}\right)$$

   have a local inverse near $(0,1)$? Justify your answer.