1. (10pts) Let f be a function defined on the interval $[0,1]$ given by $f(x) = 1$ if $x \in \mathbb{Q}$ and $f(x) = -1$ if $x \notin \mathbb{Q}$. Show that f is not Riemann integrable.

2. (15pts) Let $A = \{\frac{1}{n} \mid n \in \mathbb{N}\}$ and $B = \{\frac{1}{n} \mid n \in \mathbb{N}\} \cup \{0\}$. Answer the following questions, and *justify* your answer.

   (a) Is A closed?

   (b) Is B compact?

3. (15pts) Let $f(x) = \sqrt{x} : [0, \infty) \to \mathbb{R}$. Answer the following questions, and *justify* your answer.

   (a) Is f uniformly continuous on $[0, 2]$?

   (b) Is f uniformly continuous on $[1, \infty)$?

   (c) Is f uniformly continuous on $[0, \infty)$?

4. (15pts) Let $f_n : [0,1] \to [0, \infty)$ be a sequence of continuous functions pointwise convergent to zero, i.e., $f_n \to 0$ pointwise. Prove or disprove that

$$\lim_{n\to\infty} \int_0^1 f_n(x)dx = \int_0^1 \lim_{n\to\infty} f_n(x)dx.$$

5. (15pts) A real-valued function defined on (a,b) is called *convex* when the following inequality holds for $x, y \in (a,b)$ and $t \in [0,1]$:

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y).$$

   If f has a continuous second derivative and $f'' > 0$, show that f is *convex*.

6. (15pts) Show that

$$\int_0^\infty \frac{\sin x}{x}\, dx$$

   is *conditionally but not absolutely* convergent.

7. (15pts) Does the map

$$(x,y) \mapsto \left(\frac{xy}{x^2+y^2}, \frac{x^2-y^2}{x^2+y^2}\right)$$

   have a local inverse near $(0,1)$? Justify your answer.