

Show all your work.

- (10 pts.) State the Mean Value Theorem and show that  $|\sin x - \sin y| \leq |x - y|$  for all  $x$  and  $y$  by applying the Mean Value Theorem.
- (10 pts.) Assume that a spherical snowball melts in such a way that its radius decreases at a constant rate. Suppose it begins as a sphere with radius 10 cm and takes 2 hours to disappear. What is the rate of its volume after 1 hour? At what rate is its surface area changing after 1 hour?
- (10 pts.) Sketch the graph of  $f(x) = x^{2/3}(2x + 5)$ , and determine as many as possible of the key features such as range, intercepts, relative extrema, inflection points, asymptotes, and concavity.
- (10 pts) Find the dimensions of the right circular cylinder of largest volume that can be inscribed in a right circular cone of radius  $R$  and altitude  $H$ .
- (10 pts) The rate at which an epidemic spread through a community is proportional to the product of the number of residents who have been infected and the number of susceptible uninfected residents. Express the number of residents who have been infected as a function of time and sketch the graph of the function.
- (10 pts) Find the area of the surface generated by revolving about the  $x$ -axis the top of the cardioid  $r = 1 + \cos \theta$ ,  $0 \leq \theta \leq \pi$ .
- (10 pts.) Show that the series  $\sum_{k=1}^{\infty} \frac{\ln k}{k^q}$  converges if  $q > 1$  and diverges if  $q \leq 1$ .
- (10 pts.) Let  $f(x, y) = \begin{cases} y \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ , is  $f$  continuous at  $(0, 0)$ ? Show it.
- (10 pts.) Find the absolute extrema of the function  $f(x, y) = e^{x^2 - y^2}$  over the disk  $x^2 + y^2 \leq 1$ .
- (10 pts.) Find the volume of the solid bounded below by  $z = 2x^2 + y^2$  and above by  $z = 6 - x^2 - y^2$ .