(10%) 1. If $Y \sim U(0,5)$, what is the probability that the roots of the equation $4x^2 + 4xY + Y + 2 = 0$ are both real?

(15%)2. The moment generating function of X is given by $M_X(t) = \exp(2e^t - 2)$ and that of Y by $M_Y(t) = (\frac{3}{4}e^t + \frac{1}{4})^{10}$. If X and Y are independent, compute the following questions

(5%)(a) $P\{X + Y = 2\}$,

(5%)(b) $P\{XY = 0\}$,

(5%)(c) $E(XY)$.

(15%)3. X_1 and X_2 are independent $N(0,\sigma^2)$ random variables.

(8%)(a) Find the joint distribution of Y_1 and Y_2, where

$$Y_1 = X_1^2 + X_2^2 \text{ and } Y_2 = \frac{X_1}{\sqrt{Y_1}}.$$

(7%)(b) Show that Y_1 and Y_2 are independent, and interpret this result geometrically.

(15%)4. Let $X_1,...,X_n$ be a random sample from the pdf $f(x \mid \mu) = e^{-(x-\mu)}$, where $x > \mu$, $-\infty < \mu < \infty$.

(7%)(a) Find a complete sufficient statistic for μ.

(8%)(b) Show the statistic in (a) and sample variance, S^2, are independent.

(10%)5. Let $X_1, \ldots, X_n$ be a random sample from a population with pdf

$$f(x \mid \theta) = \frac{1}{2\theta}, \quad -\theta < x < \theta, \quad \theta > 0.$$

Find, if one exists, the UMVUEof θ.

(10%)6. Let $Y_1 < Y_2 < ... < Y_n$ be the order statistics of a random sample of size n from $U(\theta - \rho, \theta + \rho)$. Find the MLEs for θ and ρ.

(15%)7. Let $X_1,\ldots,X_n$ be a random sample from a $N(\theta,\sigma^2)$ population, where σ^2 is known. Consider testing

$$H_0:\theta\le\theta_0 \text{ versus } H_1:\theta>\theta_0.$$

(8%) (a) Find a size α likelihood ratio test.

(7%) (b) Find a UMP level α test.

(10%)8. Let $X_1,\ldots,X_n$ be a random sample from $N(\theta,\theta)$ with an unknown $\theta>0$. Find a pivotal quantity and use it to construct a $1-\alpha$ confidence interval for θ.