

1. (7 points) Use the limit definition of differentiation to find $\frac{df}{dx}$:

$$f(x) = \sqrt{x^2 + 1}.$$

2. (7 points) Find $\frac{dy}{dx}$ by implicit differentiation for

$$e^{xy} = 5x + 4y^{1/3} + 3.$$

3. (7 points) Find the limit

$$\lim_{x \rightarrow 0} \sqrt[5]{x} \sin \frac{1}{x^2}.$$

4. (7 points) Compute the integral

$$\int_0^1 x \sqrt[3]{1-x} dx.$$

5. (7 points) Compute the integral

$$\int e^x \cos 2x dx.$$

6. (7 points) Compute the integral

$$\int \frac{1}{(x-1)(2x+1)} dx.$$

7. (7 points) Find the area of the region bounded between the curve $y = x^2 - x$, and the x -axis, for $x \in [-1, 3]$.

8. (7 points) Find the limit

$$\lim_{x \rightarrow 0^+} (\sin x)^x.$$

9. (7 points) Determine whether the series converges or diverges:

$$\sum_{k=2}^{\infty} \frac{2k+1}{\sqrt{k^4-1}}.$$

10. (7 points) Let $f(x, y) = x^3 + y^4 - 6x^2 + 4y - 4$. Find the stationary points and classify the stationary points as local maximum, local minimum or saddle point.

11. (7 points) Evaluate the double integral

$$\iint_{\Omega} (xy - y^2) dx dy$$

where Ω is the region bounded by the curves $y = 0$, $x = y$, $y = 1$, and $x = -1$.

12. (7 points) Find a function $f(x, y)$ given that

$$\frac{\partial f}{\partial x} = 4x^3y^3 - 3x^2, \quad \frac{\partial f}{\partial y} = 3x^4y^2 + \cos 2y.$$

13. (16 points) Let $f(x) = x^3 - 9x^2 + 24x - 7$.

- (a) Find the intervals on which the graph of f is increasing or decreasing.
(b) Find the intervals on which the graph of f is concave upward or concave downward.
(c) Find all local maximum, local minimum and points of inflection.
(d) Sketch the graph of the function $f(x)$.