Attentions!! In the following problems, all matrices have entries in $\mathbb{R}$.

(1) (30 points) Let

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

(a) Diagonalize A. In other words, find a matrix P such that $P^{-1}AP$ is a diagonal matrix.
(b) Find A^{35}.

(2) (25 points) Find the Jordan form of

$$\begin{pmatrix} 2 & -1 & 0 & 1 \\ 0 & 3 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 3 \end{pmatrix}.$$

(3) (20 points) Let A be a square matrix of size $n \geq 2$. Show that $\det(\operatorname{adj} A) = (\det A)^{n-1}$.

(4) (25 points) Let V be an $\mathbb{R}$-vector space. Let $T\colon V \to V$ be an R-linear transformation with the property $T^2 = T$.
(a) Let $U = \{v \in V : T(v) = v\}$. Show that U is a subspace of V.
(b) Let A be a basis for the null space of T, and let B be a basis for the subspace U in (a). Show that $A \cap B = \varnothing$ and $A \cup B$ is a basis for V over $\mathbb{R}$.