國立中正大學 106 學年度碩士班招生考試試題

系所別：數學系　　　　　　　　　　　　科目：高等微積分

1. Find (i) (6pts) $\frac{d}{dx}(1+\sin x)^{1/x}$, (ii) (7pts) $\lim_{x\to 0}(1+\sin x)^{1/x}$, and (iii) (7pts) $\frac{d}{dx}\int_{x^2}^{\sin x} e^{x^2}\,dx$.

2. (20pts) Let
$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$
Find and discuss the continuity for $f'(x)$.

3. (20pts) Let $f : S \to S$ be a function from a metric space (S, d). The function $f(x)$ is called a contraction on S if there is a positive constant $c < 1$ such that
$$d(f(x), f(y)) \leq cd(x, y) \text{ for all } x, y \in S.$$
Claim that a constraction f of a complete metric space (S, d) has a unique fixed point $p \in S$.

4. (a) (10pts) Compute the second-order Taylor formula for $f(x, y) = \cos(2x + y)$ around $(0, 0)$.

   (b) (10pts) Verify $f_{xy} = f_{yx}$ for
$$f(x, y) = xy^2 \tan^{-1}(x + y).$$

5. (20pts) Show that the improper integral
$$\int_{2\pi}^{\infty} \frac{\sin x}{x} dx$$
is conditionally but not absolutely convergent.