

Abstract Algebra (II), Midterm Exam I

March 25, 2008

Instruction: Show your reasoning. Answers without explanation may receive no credit. The starred problems are a bonus; they provide extra credit. Points total 110 + 10.

1. (20 pts) Do either (a) or (b), but not both.
 - (a) Prove that a group of order 455 is abelian.
 - (b) Classify groups of order 45.
 2. (20 pts) Suppose that G is a group of order 140 and G has a normal subgroup A of order 2. Show that G has normal subgroups of order 10, 14, and 70.
 3. (30+5 pts)
 - (a) Show that any finite field has order p^n for some prime p .
 - (b) Show that $\mathbb{Z}_3[i]$ is a field of order 9.
 - (c) Is $\mathbb{Z}_5[i]$ a field? Why or why not?
 - ***(d)** Construct a field of order 25. Explain your answer.
 4. (40+5 pts)
 - (a) There is an integral domain of order 10.
 - (b) Let R be commutative and have prime characteristic p and $a, b \in R$. Then $(a + b)^p = a^p + b^p$.
 - (c) Suppose that R is a ring in which $a^2 = a$ for all a in R . Then R has characteristic 2.
 - (d) The group of units of $\mathbb{Z}[i]$ have order 4.
 - (e) The group of units of $\mathbb{Z}[x]$ have order 2.
 - ***(f)** The group of units of $\mathbb{Z}[\sqrt{2}]$ have infinite order.
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