1. (12 pts.) Find the inverse matrix of

\[ A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 1 \end{bmatrix} \]

2. (12 pts.) Consider the following \( n \times n \) tridiagonal matrix:

\[ A_n = \begin{bmatrix} 1 & 1 & 0 & 0 & \cdots & \cdots & \cdots \\ -1 & 1 & 1 & 0 & 0 & \cdots & \cdots \\ 0 & -1 & 1 & 1 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & 0 -1 1 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & 0 -1 1 \end{bmatrix} \]

Find \( \det(A_n) \).

3. (12 pts.) Let \( A \) be a \( 3 \times 3 \) matrix with eigenvalues -1, 0, and 1. Compute the determinant of the matrix \( A^3 + 2A^2 + 3I \) where \( I \) is the \( 3 \times 3 \) identity matrix.

4. (12 pts.) Let \( A \) and \( B \) be two \( n \times n \) matrices with \( AB = BA \). If all of the eigenvalues of \( A \) are real and distinct, show that \( B \) is diagonalizable.

5. (12 pts.) Let \( A \) be an \( n \times n \) symmetric matrix, and let \( \text{Col}A \) and \( \text{Nul}A \) be the column space and null space of \( A \) respectively.
   (1) Show that \( (\text{Col}A)^\perp = \text{Nul}A \).
   (2) Show that each \( x \) in \( \mathbb{R}^n \) can be written in the form \( x = y + z \), with \( y \) in \( \text{Col}A \) and \( z \) in \( \text{Nul}A \).

6. (40 pts.) True or False, with reason if true and counterexample if false.
   (1) Suppose \( u, v, w \) are nonzero vectors in \( \mathbb{R}^5 \), \( v \) is not a multiple of \( u \), and \( w \) is not a linear combination of \( u \) and \( v \). Then \( \{u, v, w\} \) is linearly independent.
   (2) If \( P \) is an invertible \( m \times m \) matrix, then \( \text{rank } PA = \text{rank } A \).
   (3) If \( A \) is row equivalent to the identity matrix \( I \), then \( A \) is diagonalizable.
   (4) A square matrix \( A \) is invertible if and only if there is a coordinate system in which the transformation \( x \rightarrow Ax \) is represented by a diagonal matrix.
   (5) If \( W \) is a subspace of \( \mathbb{R}^n \), then \( W \) and \( W^\perp \) have no vectors in common.
   (6) The equation \( Ax = b \) has a solution if and only if \( b \) is orthogonal to all solutions of the equation \( A^\top x = 0 \).
   (7) The dimension of an eigenspace of a symmetric matrix equals the multiplicity of the corresponding eigenvalue.
   (8) Every \( n \)-dimensional vector space is isomorphic to \( \mathbb{R}^n \).