Write down your answer as clear as possible, each problem is of 20 points.

1. Give a definition of upper semi-continuous (u.s.c.) functions; prove or disprove the following statements
   (a) If \( f, g \) are u.s.c. then \( f + g \) is u.s.c.
   (b) If \( \{ f_n \} \) is a sequence of non-negative u.s.c. functions so is \( \sum_{n=1}^{\infty} f_n \)

2. (a) Let \( f(x) = ax + b \), show that if \( E \) has measure \( \lambda \) then \( f(E) \) has measure \( \lambda \) a.
   (c) Let \( f(x) = x^2 + 2x \), show that if \( E \) has measure 0 so is \( f(E) \)

3. (a) Show that if \( \mu(X) < \infty, 0 < p < q < \infty \), then \( L^q \subseteq L^p \).
   (b) If \( 0 < r < p < s < \infty \), show that \( L^r \cap L^s \subseteq L^p \)

4. If \( f \) is of bounded variation on \( [a, b] \), show that \( \int_a^b |f'| \leq V[a, b] \). Show that if the equality holds in this inequality, then \( f \) is absolutely continuous.

5. (a) Let \( X \) be a compact metric space. Show that given any open cover \( \{ O_a \} \), there exits \( \delta > 0 \) such that any subset of diameter less than \( \delta \) is contained in some \( O_a \)
   (b) Let \( X, Y \) be metric spaces with \( X \) is compact. Show that every continuous function from \( X \) to \( Y \) is uniformly continuous.