1. (15%) Let \( f \) be a continuous function from \([0, 1]\) to \( R \). Show that
\[
\exp \left( \int_0^1 f(t) dt \right) \leq \int_0^1 \exp(f(t)) dt,
\]
where \( \exp \) is defined by
\[
\exp(x) = e^x, \quad x \in R.
\]

2. (25%) Let
\[
l_2 = \left\{ < a_n >_{n=1}^{\infty} \mid \sum_{n=1}^{\infty} |a_n|^2 < \infty, \ a_n \in R, \ n \geq 1 \right\}.
\]
Define the function \( d : l_2 \times l_2 \to R^+ \), by
\[
d(a, b) = \left( \sum_{n=1}^{\infty} |a_n - b_n|^2 \right)^{\frac{1}{2}},
\]
for all \( a, b \in l_2 \), with
\[
a = < a_n >_{n=1}^{\infty}, \quad b = < b_n >_{n=1}^{\infty}.
\]
Show that \( < l_2, d > \) is a complete metric space. Then show that it is also separable.

3. (15%) Let \( f \in L^1(R^n) \) and \( \hat{f} \) be the Fourier transform of \( f \).
   
   (a) Show that
   \[
   \lim_{|\xi| \to \infty} \hat{f}(\xi) = 0.
   \]

   (b) For \( g \in L^2(R^n) \), is it true that
   \[
   \lim_{|\xi| \to \infty} \hat{g}(\xi) = 0? 
   \]
   You shall give the reason.

4. (15%) Let \( f, g \in C_0^\infty(R) \), and \( h \) be defined by
\[
h(x) = \int_{-\infty}^{\infty} f(x-y)g(y)dy, \quad x \in R.
\]
Show that
\[
(\partial_x h)(x) = \int_{-\infty}^{\infty} (\partial_x f)(x-y)g(y)dy, \quad x \in R.
\]

5. (30%) Let \( f \) and \( g \) be two functions from \( R \) to \( R \). Suppose \( f \) is continuous in \( R \) and \( \text{Supp } f \) is compact. Meanwhile, \( g \) is of Bounded variation in \( R \).
   
   (a) Show that the Riemann-Stiejes integral
   \[
   \int_{-\infty}^{\infty} f dg \text{ exists.}
   \]

   (b) Find a Borel measure \( \mu \) so that
   \[
   \int_{a}^{b} f dg = \int_{a}^{b} f du,
   \]
   where \([a, b]\) is any closed interval in \( R \) with \( a < b \).